

Claims

1. A method of securely implementing a public-key cryptography algorithm, the public key being composed
 5 of an integer n that is a product of two large prime numbers p and q , and of a public exponent e , said method consisting in determining a set E comprising a predetermined number of prime numbers e_i that can correspond to the value of the public exponent e , said
 10 method being characterized in that it comprises the following steps consisting in:

a) computing a value $\Phi = \prod_{e_i \in E} e_i$

such that Φ/e_i is less than $\Phi(n)$ for any e_i
 15 belonging to E , where Φ is the Euler totient function;

b) applying the value Φ to a predetermined computation;

c) for each e_i , testing whether the result of said predetermined computation is equal to a value Φ/e_i :

20 - if so, then attributing the value e_i to e , and storing e with a view to it being used in computations of said cryptography algorithm;

- otherwise, observing that the computations of the cryptography algorithm using the value e cannot be
 25 performed.

2. A method according to claim 1, characterized in that the cryptography algorithm is based on an RSA-type algorithm in standard mode.

3. A method according to claim 2, characterized in that the predetermined computation of step b) consists in computing a value C:

5 $C = \Phi \cdot d \text{ modulo } \Phi(n)$, where d is the corresponding private key of the RSA algorithm such that $e \cdot d = 1 \text{ modulo } \Phi(n)$ and Φ is the Euler totient function.

10 4. A method according to claim 2, characterized in that the predetermined computation of step b) consists in computing a value C:

$C = \Phi \cdot d \text{ modulo } \Phi(n)$, where d is the corresponding private key of the RSA algorithm such that $e \cdot d = 1 \text{ modulo } \Phi(n)$, with Φ being the Carmichael function.

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 5. A method according to claim 1, characterized in that the cryptography algorithm is based on an RSA-type algorithm in CRT mode.

20 6. A method according to claim 5, characterized in that the predetermined computation of step b) consists in computing a value C:

25 $C = \Phi \cdot d_p \text{ modulo } (p-1)$, where d_p is the corresponding private key of the RSA algorithm such that $e \cdot d_p = 1 \text{ modulo } (p-1)$.

 7. A method according to claim 5, characterized in that the predetermined computation of step b) consists in computing a value C:

$C = \Phi \cdot d_q \text{ modulo } (q-1)$, where d_q is the corresponding private key of the RSA algorithm such that $e \cdot d_q = 1 \text{ modulo } (q-1)$.

5 8. A method according to claim 5, characterized in that the predetermined computation of step b) consists in computing two values C_1 and C_2 such that:

$C_1 = \Phi \cdot d_p \text{ modulo } (p-1)$, where d_p is the corresponding private key of the RSA algorithm such
10 that $e \cdot d_p = 1 \text{ modulo } (p-1)$;

$C_2 = \Phi \cdot d_q \text{ modulo } (q-1)$, where d_q is the corresponding private key of the RSA algorithm such that $e \cdot d_q = 1 \text{ modulo } (q-1)$;

and in that the test step c) consists, for each
15 e_i , in testing whether C_1 and/or C_2 is equal to the value Φ/e_i :

- if so, then attributing the value e_i to e and storing e with a view to it being used in computations of said cryptography algorithm;

20 - otherwise, observing that the computations of said cryptography algorithm using the value e cannot be performed.

9. A method according to claim 3 or claim 4 and
25 in which a value e_i has been attributed to e , said method being characterized in that the computations using the value e consist in:

choosing a random integer r ;

computing a value d^* such that $d^* = d + r \cdot (e \cdot d - 1)$;
and

implementing a private operation of the algorithm
in which a value x is obtained from a value y by
5 applying the relationship $x = y^{d^*}$ modulo n .

10. A method according to any one of claims 2 to
4, and in which a value e_i has been attributed to e ,
said method being characterized in that it consists,
10 after a private operation of the algorithm, in
obtaining a value x from a value y , and in that the
computations using the value e consist in checking
whether $x^e = y$ modulo n .

15 11. A method according to any one of claims 5 to
8, and in which a value e_i has been attributed to e ,
characterized in that it consists, after a private
operation of the algorithm, in obtaining a value x from
a value y , and in that the computations using the value
20 e consist in checking firstly whether $x^e = y$ modulo p
and secondly whether $x^e = y$ modulo q .

12. A method according to any preceding claim,
characterized in that the set E comprises at least the
25 following e_i values: 3, 17, $2^{16} + 1$.

13. An electronic component characterized in that
it comprises means for implementing the method
according to any preceding claim.

14. A smart card including an electronic component according to claim 13.

15. A method of securely implementing a public-key cryptography algorithm, the public key being composed of an integer n that is a product of two large prime numbers p and q , and of a public exponent e , said method consisting in determining a set E comprising a predetermined number of prime numbers e_i that can correspond to the value of the public exponent e , said method being characterized in that it comprises the following steps consisting in:

a) choosing a value e_i from the values of the set E ;

b) if $\Phi(p) = \Phi(q)$, testing whether the chosen e_i value satisfies the relationship:

$$(1 - e_i \cdot d) \text{ modulo } n < e_i \cdot 2^{(\Phi(n)/2) + 1}$$

or said relationship as simplified:

$$(-e_i \cdot d) \text{ modulo } n < e_i \cdot 2^{(\Phi(n)/2) + 1}$$

where $\Phi(p)$, $\Phi(q)$, and $\Phi(n)$ are the functions giving the numbers of bits respectively encoding the number p , the number q , and the number n ;

otherwise, when p and q are unbalanced, testing whether the chosen e_i value satisfies the following relationship:

$$(1 - e_i \cdot d) \text{ modulo } n < e_i \cdot 2^{g+1}$$

or said relationship as simplified:

$$(-e_i \cdot d) \text{ modulo } n < e_i \cdot 2^{g+1}$$

with $g = \max(\Phi(p), \Phi(q))$, if $\Phi(p)$ and $\Phi(q)$ are known, or, otherwise, with $g = \Phi(n)/2 + t$, where t designates the imbalance factor or a limit on that factor;

5 c) if the test relationship applied in the preceding step is satisfied and so $e = e_i$, storing e with a view to using it in computations of said cryptography algorithm;

10 - otherwise, reiterating the preceding steps while choosing another value for e_i from the set E until an e_i value can be attributed to e and, if no e_i value can be attributed to e , then observing that the computations of said cryptography algorithm using the value of e cannot be performed.

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16. A method according to claim 15, characterized in that, for all values of i , $e_i \leq 2^{16} + 1$, and in that the step b) is replaced by another test step consisting in:

20 b) if $\Phi(p) = \Phi(q)$, testing whether the chosen e_i value satisfies the relationship:

$$(1 - e_i \cdot d) \bmod n < e_i \cdot 2^{(\Phi(n)/2) + 17}$$

or said relationship as simplified:

$$(-e_i \cdot d) \bmod n < e_i \cdot 2^{(\Phi(n)/2) + 17}$$

25 where $\Phi(p)$, $\Phi(q)$, and $\Phi(n)$ are the functions giving the numbers of bits respectively encoding the number p , the number q , and the number n ;

otherwise, when p and q are unbalanced, testing whether the chosen e_i value satisfies the following relationship:

$$(1-e_i.d) \text{ modulo } n < e_i.2^{g+17}$$

or said relationship as simplified:

$$(-e_i.d) \text{ modulo } n < e_i.2^{g+17}$$

5 with $g = \max(\Phi(p), \Phi(q))$, if $\Phi(p)$ and $\Phi(q)$ are known, or, otherwise, with $g = \Phi(n)/2 + t$, where t designates the imbalance factor or a limit on that factor.

10 17. A method according to claim 15, characterized in that step b) is replaced with another test step consisting in:

testing whether the chosen e_i value satisfies the relationship whereby:

15 the first most significant bits of $(1-e_i.d)$ modulo n are zero;

or said relationship as simplified whereby:

the first most significant bits of $(-e_i.d)$ modulo n are zero.

20 18. A method according to claim 17, characterized in that the test is performed on the first 128 most significant bits.

25 19. A method according to any one of claims 15 to 18, characterized in that the cryptography algorithm is based on an RSA-type algorithm in standard mode.

20. A method according to any one of claims 15 to 19, and in which an e_i value has been attributed to e ,

said method being characterized in that the computations using the value e consist in:

- choosing a random integer r ;
- computing a value d^* such that $d^* = d + r \cdot (e \cdot d - 1)$;

implementing a private operation of the algorithm in which a value x is obtained from a value y by applying the relationship $x = y^{d^*}$ modulo n .

21. A method according to any one of claims 15 to 19 and in which an e_i value has been attributed to e , said method being characterized in that it consists, after a private operation of the algorithm, in obtaining a value x from a value y and in that the computations using the value e consist in checking whether $x_e = y$ modulo n .

22. A method according to any one of claims 15 to 21, characterized in that the set E comprises at least the following e_i values: 3, 17, $2^{16}+1$.

23. A method according to claim 22, characterized in that the preferred choice of the values e_i from the values of the set E is made in the following order: $2^{16}+1$, 3, 17.

24. An electronic component characterized in that it comprises means for implementing the method according to any one of claims 15 to 23.

25. A smart card including an electronic component according to claim 24.